Lewin: K-nets and GIS

In his article on Klumpenhouwer networks (or K-nets), David Lewin explains how to represent intervallic relations within trichords and tetrachords via a network structure. These networks, named after his student Henry Klumpenhouwer, show how each note of a pitch-set can transpose or invert into the other notes of the collection; Lewin uses a standard T_n/I_n labeling system for this purpose. When two sets of notes display similar transpositional and inversional characteristics, Lewin asserts that, due to their similar network structures, they are therefore *isographic*. He goes on to explore different types of isography using concrete examples from the music repertoire, while also making these isographies explicit through mathematical proofs and definitions. Furthermore, Lewin develops *recursive* structures, i.e. networks-of-networks, to demonstrate hierarchical levels of organization based on transformational theory.

Lewin begins his discussion of K-nets by giving some "basic properties." In his Figure 2, he shows how multiple and different networks can be constructed to describe the transformational characteristics of a single trichord. Lewin is thus already implicitly evoking the role of the theorist in not simply labeling features of the music, but rather in organizing and shaping the musical content of a work via analysis in order to create a more unified picture of the whole. When the theorist has organized two trichords such that their network structures are identical, Lewin calls these networks *strongly isographic*. This strongly isographic relationship allows theorists to relate trichords of different set-classes that do not share the exact same intervallic content. Trichords of the same set-class, however, even those that are literal transpositions of one another, are not strongly isographic. In other words, a trichord under transposition will not have an identical network structure as the original. Instead of isographic, Lewin terms this relationship a *network isomorphism*, meaning the two trichords have the same shape (but slightly different graphs due to transposition).

If two trichords are strongly isographic and then one of the trichords is transposed, the two trichords lose their strong isography. Yet all that has occurred is that one of the trichords has been transposed (via a network isomorphism). Since the intervallic structure of the trichords has remained the same, this transposed trichord must still relate back somehow to its original strongly-isographic partner. Lewin thus gives four more categories of isography, all of which describe cases where one trichord of a strongly isographic pair has undergone some sort of transformation. In the case of transposition (Rule 1), Lewin calls the relation *positive isography*. He uses the notation <1,j> to describe positive isography between trichords, with *j* representing the amount of transposition of the second trichord away from a strongly-isographic state with the first trichord. In the accompanying case of inversion (Rule 2), Lewin uses the notation <11,j> and calls this *negative isography*, with *j* representing the level of inversion. Thus, if one trichord of a strongly isographic pair of trichords undergoes an I_4 transformation, for instance, Lewin labels the relation of the new trichord to its original strongly isographic partner as <11,4>. Isographies under transformations via M_5 and M_7 are also defined (<5,j> and <7,j> respectively), but Lewin does not go into much more detail or provide further examples of such M-relations.

In order to fully expose just how "flexible and powerful" K-net resources are, Lewin shifts to an analysis of certain aspects from a small section of Schoenberg's *Pierrot Lunaire* No. 4. In this analysis, Lewin carefully labels each network he builds out of notes from mm. 13-14 in the piece. He then proceeds to derive the <1,j> or <11,j> relations between each graph. The crucial step for Lewin is creating an analogy between <1,j>/<11,j> isographies and T_n/I_n transformations, because once this analogous function is understood, Lewin can create Klumpenhouwer networks out of the <1,j> and <11,j> isographies themselves. One culmination for this line of thinking is Lewin's Figure 11, where he shows how the network structure of a local trichord maps exactly to the network structure of the local networks themselves. In this example, Lewin is also careful to point out that much of the ability to create such low- and high-level relationships stems from the flexibility of K-nets to interpret pitch-class sets in a variety of ways, thereby allowing the theorist to mold this organization to his or her needs. In a further example taken from this piece (Example 12), Lewin again demonstrates how higher hierarchical levels of K-nets "prolong" structures of the trichords themselves, this prolongation being akin to the prolongation of tonic and dominant harmonies in tonal music.

This "recursive" nature of Klumpenhouwer networks undergoes further exploration in the article. First, Lewin proves the mathematical validity of relating <1,j> and <11,j> functions to T_n/I_n transformations. He allows for a limitless extension of networks-of-networks, adding that isographies may be applied between any level of the system and another. Again, the reader is reminded that the chords of the music do not inherently give network structures, but that the theorist must construct the graphs as interpretations. Furthermore, configuring such interpretations is not an "automatic affair" but rather "a combination of art and will".

To reinforce the role of the theorist in developing networks that elucidate the most salient musical relationships in a work, Lewin returns to a musical analysis of *Pierrot Lunaire*. In this example, Lewin seeks to show how the opening harmonies of a particular phrase elaborate (through their network structure) an important augmented triad sonority that closes the phrase. Lewin picks between positive and negative isographic representations of the harmonies to better bolster his theoretical case. Figure 16 is contrasted against Figure 15; while both interpretations develop recursive networks based on the structure of the augmented triad, the latter shows the elaboration of characteristics more particular to the augmented triad than Figure 16. Thus, Lewin has implied that certain recursive structures are more useful and theoretically revealing than others.

In the final and lengthy analytical section of his paper, Lewin delves into another phrase from *Pierrot Lunaire* to demonstrate the use of tetrachords arranged in Klumpenhouwer networks. Since tetrachords (like trichords) can be interpreted in a variety of different ways by isographic networks, Lewin looks separately at four distinct interpretations (called "MODE"s). MODE I and MODE II analyze tetrachords based on finding a common trichord within each. A parachute-like graph results from this common tetrachord. Lewin is able to preserve his <1,j> and <11,j> relationships between these tetrachordal networks, thus allowing him to develop higher-level parachute-like tetrachordal networks such as his Figure 19. For his MODE III and MODE IV analyses, Lewin extracts two common dyads out of each tetrachord. The result is a circular-type network graph such as is shown in Figure 26. Again, <1,j> and <11,j> relationships persist, allowing higher-level circular-type network graphs to describe the recursive network structure of this phrase. Through his variety of MODE examples, Lewin thus exhibits how a variety of Klumpenhouwer networks can show valid and hierarchical interpretations for a given piece of music. But although Lewin is careful to show how certain interpretations involving K-nets represent more organic views of the music's organization, he does not give the reader any tools with which to determine whether any one network realization is more musically valid than another, i.e. which network bests represents what either the composer intended for the work and/or the listener perceives when hearing the work.

WORKS CITED

Lewin, David. "Klumpenhouwer Networks and Some Isographies That Involve Them." *Music Theory Spectrum* 12.1 (Spring 1990): 83-120.

Table of Lewin's GIS examples (from *Generalized Music Intervals and Transformations*, 1987)

Example	**space (S)**	**IVLS**	**int(s,t)**
2.1.1	the diatonic gamut, extended indefinitely up and down (A1,B1,C1...D4...D5..etc.)	all integers (positive and negative) including zero	number of steps from *s* to *t*; positive for steps up, negative for steps down
2.1.2	chromatic scale, extended indefinitely up and down	all integers (positive and negative) including zero	number of semitones from *s* to *t*; positive for steps up, negative for steps down
2.1.3	the twelve pitch classes	integers modulo 12 (0,1,2,3,4,5,6,7,8,9,T,E)	the number of hours clockwise from *s* to *t* on a 12-hour clock
2.1.4	seven pitch classes corresponding to the diatonic gamut	integers modulo 7 (0,1,2,3,4,5,6)	the number of hours clockwise from *s* to *t* on a 7-hour clock
2.1.5	family of "pitches" available from a given pitch under just intonation	all rational numbers that can be expressed via $2^a 3^b 5^c$, where a, b, and c are integers	the frequency of *t* divided by the frequency of *s*, i.e. FQ(t)/FQ(s)
2.1.6	the "game board" of Lewin's Figure 2.2 (circle of 5ths left-right, cycle of M3s up-down)	(b, c) where *b* and *c* are integers	(b, c) where *t* lies *b* squares to the right and *c* squares above *s* on the game board
2.2.1	a succession of regularly-spaced time points pulsing "one time unit" apart"	all integers (positive and negative) including zero	the number of time units by which *t* is later than *s* (negative numbers indicating an earlier value)
2.2.2	the space of 2.2.1 wrapped around an N-hour clock (like a musical measure)	integers modulo N (0,1,2,3,...N-1)	the number of hours clockwise that *t* lies from *s* on the N-hour clock
2.2.3	a family of durations, each duration measured in positive integer values of time units	a group of positive ratios, based on the chosen rhythmic durations	the durational quotient of *t* divided by *s* (*t* units/*s* units)
2.2.4	the space of 2.2.3 reduced to equivalence classes by a durational modulus M greater than 1	the IVLS of 2.2.3 reduced to ratio-classes by powers of M	the durational quotient of *t* divided by *s*, reduced by modulus M
2.2.5	a family of durations (like 2.2.3) that are measured in units of time	all integers (positive and negative) including zero	the difference of time units between *s* and *t*; because of the potential for negative values of *t*, this example is not a GIS
2.2.6	the durations of 2.2.5 reduced by modulus M to duration-classes	integers modulo M (0,1,2,3,....M-1)	the number of hours clockwise that *t* lies from *s* on the M-hour clock